**Quadratic Functions**

The general form of a quadratic function is $f(x) = ax^2 + bx + c$ or $y = ax^2 + bx + c$.

**Basic quadratic:** $f(x) = x^2$  
The graph of any quadratic function is called a “parabola”.

In order to graph a basic quadratic function, we will use these $x$-values: $-2, -1, 0, 1,$ and $2$.

The shape we get from this graph is the general shape of all parabolas. All quadratic functions have this overall shape.

**General parabolic shapes:**

[Diagram of a parabola with labeled axis of symmetry and vertex.]

**Interesting Things About Parabolas**

1. The $y$-values start to repeat themselves at some point. (This point is called the vertex.)
2. The graph is symmetrical. The point at which we can fold our graph in half is the vertex. The “crease” down the middle of the graph is called the axis of symmetry.
3. If we find the vertex of the parabola first, the $y$-values on either side of the vertex are the same.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
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</thead>
<tbody>
<tr>
<td>-2</td>
<td>4</td>
</tr>
<tr>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
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<tr>
<td>1</td>
<td>1</td>
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<td>2</td>
<td>4</td>
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If we put these things together, we can see that if we find the vertex of our parabola first, then we can get an idea about what the rest of the data is going to look like.
because it “centers” our graph. So, that’s how we’re going to go about figuring out how
to graph parabolas in general. We’re going to find the vertex first, and then we’re going
to pick points on each side of that vertex.

Websites of Interest:
http://www.purplemath.com/modules/grphquad.htm  (There are 4 pages to this lesson.)
http://www.youtube.com/watch?v=tjroyVI8E14

How to Find the Vertex of a Parabola

Given \( f(x) = ax^2 + bx + c \):

1) The x-value of the vertex = \( \frac{-b}{2a} \)

This is the front part of the quadratic formula, and since we’ve already talked about
the set-up for how the quadratic formula works, it makes sense to do our set-up for
the vertex formula the same way:

\[
x = \frac{-b}{2a} = \frac{-b}{2(\frac{b}{2})} = \frac{-b}{b} = \frac{-b}{b}
\]

2) The y-value of the vertex \( \Rightarrow \) plug what you get for “x” into \( ax^2 + bx + c \).

Example 1: Find the vertex of the parabola given by \( f(x) = 2x^2 + 4x – 3 \).

\[
a = 2 \quad b = 4 \quad c = -3 \quad \text{Vertex} = (_______, _______)
\]

\[
x = \frac{-b}{2a} = \frac{-4}{2(2)} = \frac{-2}{2} = -1
\]

\[
y = 2(______)^2 + 4(______) – 3
\]

\[
= 2(______) + (______) – 3
\]

\[
= _____ + _____ – 3
\]

\[
= ______
\]

Example 2: Find the vertex of the parabola given by \( f(x) = 3x^2 + 12x – 5 \)

\[
a = ______ \quad b = ______ \quad c = ______ \quad \text{Vertex} = (_______, _______) 
\]

\[
x = \frac{-b}{2a} = \frac{-12}{2(3)} = \frac{-6}{6} = -1
\]

\[
y = 3(______)^2 + 12(______) – 5
\]

\[
= _____ + _____ – 5
\]

\[
= ______
\]
Practice – Find the vertex.
1) \( f(x) = 2x^2 - 8x + 9 \) 
2) \( f(x) = x^2 + 10x - 15 \)

Websites of Interest:
http://www.purplemath.com/modules/grphquad2.htm
http://www.wtamu.edu/academic/anns/mps/math/mathlab/col_algebra/col_alg_tut34_quadfun.htm

Vertex Form of a Quadratic Equation
Find the vertex in each of the parabolas below. In the second equation, multiply out the square and simplify first. What do you notice about the two equations and their vertices?

\[
f(x) = x^2 + 4x + 1 \quad \text{and} \quad f(x) = (x + 2)^2 - 3 = (x + 2)(x + 2) - 3 \quad \text{Keep going!}
\]

While the first equation is written in standard form, the second equation is written in a form called vertex form. Can you see why?

Some people get confused by the fact that the x-value of the vertex is the opposite sign of the number in the parentheses. The question we need to be asking is this:

“What number do we plug in for ‘x’ to make the parentheses equal to zero?”

Once you get “x”, then plug it in to get “y”. (The “y” sign isn’t the opposite, unlike the “x” sign.)

What are the vertices of the following?

\[
f(x) = (x - 3)^2 + 5 \quad \text{and} \quad f(x) = 3(x + 2.5)^2 - 7.3
\]

Note: Many books refer to \( y = a(x - h)^2 + k \) as standard form instead of vertex form.

Practice – Find the vertex of the following parabolas.
1) \( f(x) = 3(x - 4)^2 + 6 \) 
2) \( f(x) = -2(x + 9)^2 - 5 \) 
3) \( f(x) = 4(x + 0.6)^2 + 0.43 \)
How to Graph a Parabola

1. Find the vertex. (Remember, finding the vertex first helps to center your graph!)

2. For the other x-values, pick the previous two integers and next two integers with respect to the x-value of the vertex.

3. Plug each of those x-values into the equation to get the corresponding y-values.

4. Plot your points!

Example 1: Graph the parabola given by \( f(x) = x^2 + 6x + 8 \).

1) Find the vertex. \( x = \frac{-b}{2a} = \frac{-6}{2} = -3 \)

\[
y = \left( \frac{-3}{2} \right)^2 + 6(\frac{-3}{2}) + 8
\]

\[
= \frac{9}{4} - 9 + 8
\]

\[
= \frac{1}{4}
\]

2) Pick the x-values as integers above and below the x-value of the vertex.

<table>
<thead>
<tr>
<th>x</th>
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3) Plug in those x-values to get the corresponding y-values for the chart above.

\[
f(x) = x^2 + 6x + 8 \quad \text{Set-up: } y = \left( \frac{-3}{2} \right)^2 + 6(\frac{-3}{2}) + 8
\]

\[
x = \_ \quad x = \_ \quad x = \_ \quad x = \_
\]

4) Plot the points:

Websites of Interest:

http://www.purplemath.com/modules/grphquad.htm
http://www.youtube.com/watch?v=9ltJFO8cEu4

(There are 4 pages to this lesson.)
Graphing Parabolas in Vertex Form

Remember, when we’re graphing a parabola, we want to find the vertex first, and then find two other points on either side of the vertex to graph so that we get the curved shape we’re all familiar with. When a quadratic equation is in vertex form, the vertex is much easier to find than if the quadratic equation is in standard form.

**Example:** Graph \( f(x) = (x - 2)^2 + 1 \)

The vertex is \((2, 1)\). Since the x-value for the vertex is “2”, then for the other x-values, we’ll pick 3 and 4 on the left and 0 and 1 on the right. (The two nearest, nice x-values to \(x = 2\)). Then, plug in 0, 1, and 3, 4 for “x” and see what we get for \(f(x)\).

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
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<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
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In a lot of ways, graphing a quadratic equation that is written in vertex form is almost easier than graphing one that is written in standard form. Plugging in the other values of x (0, 1, and 3, 4) is easier, and this form makes it a little more evident why parabolas are symmetric.

**Websites of interest:**
- [http://www.analyzemath.com/quadratics/quadratics.htm](http://www.analyzemath.com/quadratics/quadratics.htm) (Contains an interactive link!)
- [http://www.youtube.com/watch?v=-K4S0pWxmmk](http://www.youtube.com/watch?v=-K4S0pWxmmk)