**Exponents**

Dealing with positive and negative exponents and simplifying expressions dealing with them is simply a matter of remembering what the definition of an exponent is.

- A positive exponent means *repeated multiplication*.
- A negative exponent means the *opposite* of repeated multiplication, which is *repeated division*.

Let’s evaluate the following:

\[
2^3 =
\]

\[
2^2 =
\]

\[
2^1 =
\]

\[
2^0 =
\]

\[
2^{-1} =
\]

\[
2^{-2} =
\]

\[
2^{-3} =
\]

Notice that a negative exponent makes the “2” flip from \( \frac{2}{1} \) raised to some power over to \( \frac{1}{2} \) raised to some power. In other words, instead of multiplying by 2 repeatedly, we’re dividing by two repeatedly. In effect, a negative exponent makes us do the *opposite* of what it initially looks like. If a number with a negative exponent is in the top (multiplication), it means to move it to the bottom (divide); If a number with a negative exponent is in the bottom (division), it means to move it to the top (multiply).

**Examples:** Simplify the following.

1) \( x^{-5} \)
2) \( \frac{1}{x^{-6}} \)
3) \( 5x^{-4} \)
4) \( \frac{5}{6x^{-2}} \)
5) \( (3x)^{-2} \)

Notice that only whatever is directly next to the exponent is affected by the exponent. In #3 and #4, the numbers 5 and 6 are not moved or otherwise changed. In #5, the entire parenthesis is affected by the exponent.

**Laws of Exponents**

At this point, the laws of exponents should be fairly well known to us. We’re going to examine where these “laws” came from. They’re just based on the definition of an exponent, which is repeated multiplication.

**Example 1:** Simplify: \( x^2 \cdot x^4 \)

**Example 2:** Simplify \( \frac{x^2}{x^6} \)
Example 3: Simplify \((x^3)^4\)  

Example 4: Simplify \(\left(\frac{x^4}{y^3}\right)^2\)

When simplifying an expression involving positive and negative exponents, it’s sometimes easier to move the base with the negative exponents before using the laws from Examples 1 and 2. That way, we don’t have to worry about double negatives and the like.

I tend to distribute first (like in Examples 3 and 4), move my negative exponents, and then do the final simplification (Examples 1 and 2).

**Practice – Simplify.**

1) \(\frac{x^{-4}}{x^5}\)  
2) \(\left(x^{-3} y^2\right)^4\)  
3) \(\left(\frac{x^{-3}}{y^2}\right)^{-6}\)

4) \(\left(\frac{2x^2 y^{-3}}{4x^{-5} y^{-8}}\right)^2\)  
5) \(\left(\frac{12x^2}{25y^{-5}}\right) \cdot \left(\frac{15y^{-8}}{16x^{-4}}\right)^2\)

**Scientific Notation**

Scientific notation is used by mathematicians, scientists, and many others whenever the numbers they are dealing with are either really, really large or really, really small. It prevents us from having to write a bunch of zeroes.

**Examples of numbers in scientific notation (and the equivalent expanded number):**

\[
\begin{align*}
6.0221415 \times 10^{23} & \quad (\text{Avogadro's Constant}) & \quad 602,214,150,000,000,000,000,000,000 \\
1.98892 \times 10^{30} & \quad (\text{mass of the sun in kg}) & \quad 1,988,920,000,000,000,000,000,000,000,000 \\
2.8 \times 10^{-5} & \quad (\text{mass of a grain of sand in g}) & \quad 0.000028 \\
2.5 \times 10^{-2} & \quad (\text{mass of a drop of water in g}) & \quad 0.025
\end{align*}
\]

Notice that the first two numbers have positive exponents on the “10”, while the last two numbers have negative exponents on the “10”. This is because the first two are very large numbers (we’re multiplying by 10 repeatedly), whereas the last two are very small numbers (we’re dividing by 10 repeatedly).
Also notice that in all four of these numbers, there is exactly one digit to the left of the decimal place: \(6.0221415 \times 10^{23}\), \(1.98892 \times 10^{30}\), \(2.8 \times 10^{-5}\), \(2.5 \times 10^{-2}\). Scientific notation is always in the form \(._\text{______} x 10^{\text{some power}}\), where there is one digit to the left of the decimal point.

So, here’s the process for changing a number into scientific notation:

1) Move the decimal point from its original position in the number to the right of the leftmost digit.

2) a) Count how many places you moved your decimal point from its original position. This number is going to be your “some power” in the format above.
   b) If the number is large, the power will be large (positive). If the number is small, the power will be small (negative).

3) Drop any extra zeroes trailing at the right-side end of your number.

To change from scientific notation to decimal number notation, we just go the opposite way, keeping in mind that

a) if the power on “10” is positive, we’re going to move the decimal point that many places in the direction that makes the number larger.

b) if the power on “10” is negative, we’re going to move the decimal point that many places in the direction that makes the number smaller.

In both cases, you will most likely have to put in zeroes as placeholders.

**Examples:** Write the following numbers in scientific notation:

1) \(33,000,000\)  
2) \(0.000058\)

Write the following numbers in standard notation:

3) \(5.26 \times 10^{12}\)  
4) \(2.9 \times 10^{-8}\)

Evaluate the following:

5) \(\frac{15 \times 10^{-6}}{10 \times 10^{-7}}\)  
6) \(\frac{0.0004 \times 0.00006}{1.2 \times 10^{15}}\)

**Adding, Subtracting, and Multiplying Polynomials**

A polynomial has variables such that the exponents attached to them are whole numbers. In other words, polynomials *don’t* have exponents that are negative, fractions, decimals, or roots. This means that polynomials don’t have variables in the denominator or under radical signs, so the domain of any polynomial equation is always “All Real Numbers”, or \((-\infty, \infty)\).

**Examples:** 1) \(5x^4\)  
2) \(4x^6 - 5x^3 + 21\)  
3) \(2.3xy^3 - 7.64x^4y^2 + 91x\)  
4) \(x^3 \sqrt{3} + 5y^{18}\)

Notice, the *coefficients* (numbers multiplied by the variables) of the variables can be as ugly as they want to be. It’s the exponents of the variables that must be 0, 1, 2, 3, 4, 5...
Non-Examples:  a) \(2y\sqrt{x}\)  
b) \(\frac{12}{x-7}\)  
c) \(\frac{x}{y^3}\)  
d) \(x^{1.5}\)  
e) \(x^{-4}\)

Why are the above not polynomials?
a)  
b)  
c)  
d)  
e)  

Adding, Subtracting, and Combining Like Terms:
Adding and subtracting polynomials involves something mathematicians refer to as “combining like terms”. Like terms in polynomials have the same variables and the same exponents attached to those variables. The coefficient in front of those variables tells us how many of those variables we have added together. (Multiplication is a shortcut for repeated addition!)

Examples: 5) \(3x^4 + 7x^4\)  
6) \(17xy^3 - 28xy^3\)  
7) \(3x^2y - 5xy^3\)

When we’re adding and subtracting polynomials, we need to be specifically careful when distributing a negative sign across a set of parentheses.

Examples:
8) Adding: \((3x^2 - 2x + 5) + (2x^2 + 8x - 7)\)  
9) Subtracting: \((3x^2 - 2x + 5) - (2x^2 + 8x - 7)\)

Multiplying – The Distributive Property: The distributive property is the result of the fact that multiplication is a shortcut for repeated addition.

Examine the following: \(2(x + 3) = (x + 3) + (x + 3) = 2x + 6\) when we combine like terms. Notice that a shorter way will work. We can multiply the “2” by both numbers inside the parentheses and will still get \(2x + 6\).

This idea works regardless of what is out front of (or, in fact, behind) the parentheses.

Examples: 10) \(3(x^2 + 6x - 5)\)  
11) \(-2x^3(5x^2 + 6x^7 - 4x)\)  
12) \(4xy(7xy^2 - 2x^3y^6)\)

Multiplying polynomials – Many times, people associate multiplying polynomials with what is commonly referred to as the “FOIL” method. This is only partially true because “FOIL” only works in specific instances where we are multiplying two binomials.

Let’s first examine something we’ve been doing since about 4th grade now: \(12 \times 13\).
When we multiply these two numbers, notice how we multiply both the 1 and the 3 in the “13” by both the 1 and the 2 in the “12”:

\[
\begin{array}{c}
12 \\
\times 13 \\
\hline
6 \\
30 \\
20 \\
100 \\
156 \\
\end{array}
\]

\[
\begin{array}{c}
12 \\
\times 13 \\
\hline
36 \\
120 \\
156 \\
\end{array}
\]

This way is fine, but the first way shows more clearly how everything on the bottom gets multiplied by everything on the top!
Try multiplying this out using FOIL: \((x + 2)(x + 3) = \) ________________________ = ___________

What if \(x\) is 10? Use FOIL here: \((10 + 2)(10 + 3) = \) ________________________ = ___________

Another way to do this is called the “box method”, which doesn’t initially look easier, but can help with organization in larger problems. For \((x + 2)(x + 3)\), the box would look like this:

```
  x   2
   x
   3
```

Notice how we place our numbers in such a way that top (or side) is the entire first parentheses and the side (or top) is the entire second parentheses. You can put “+” signs to designate the values are positive, if you wish. When we have a negative value, we must certainly put the negative signs there.

Then we just multiply the sides of the four inner boxes and add up what we get on the inside of the boxes for the final answer, like below.

```
  x   2
x   x^2  2x
3   3x  6
```

Answer = \(x^2 + 2x + 3x + 6\)

= \(x^2 + 5x + 6\)

This works great for larger problems like \((2x + 3)(5x^2 - 7x + 9)\) because it keeps things organized.

```
  5x^2  -7x  9
2x
3
```

Again, the 2x and 3 are multiplied by everything in the second parentheses \((5x^2, -7x, \text{ and } 9)\). Our like terms are still diagonal from one another, and we can’t accidentally forget to multiply anything, so those are two more reasons this method is nice.
Practice – Box Method
Given the following labeled boxes, write the multiplication problem being demonstrated and find the final product.

1) \[\begin{array}{cc}
2x & -5 \\
4x & \\
-13 & \\
\end{array}\] 

Problem________________________
Product ________________________

2) \[\begin{array}{cc}
3 & -7x \\
6x & \\
-11 & \\
\end{array}\] 

Problem____________________________
Product _____________________________

3) \[\begin{array}{ccc}
x^2 & -4x & 9 \\
2x & & \\
-5 & & \\
\end{array}\] 

Problem___________________________
Product ____________________________

4) \[\begin{array}{ccc}
5x^3 & 3x^2 & -12x \\
x^2 & & \\
-2x & & \\
\end{array}\] 

Problem____________________________
Product ______________________________

5) \[\begin{array}{ccc}
x^2 & 3x & 7 \\
2x^2 & & \\
-4x & & \\
6 & & \\
\end{array}\] 

Problem____________________________
Product ______________________________

6) \[\begin{array}{ccc}
3x^2 & -2x & -5 \\
4x^2 & & \\
-6x & & \\
-9 & & \\
\end{array}\] 

Problem____________________________
Product ______________________________