Factors

A factor is a number that goes into another number. For example, 1, 2, 3, 6 are all factors of 6. The reason we are interested in factors is it will help us to simplify fractions into lowest terms.

There are two types of factors we’re going to be interested in.

1. Prime Numbers: Have exactly two factors, the numbers 1 and the number itself.
2. Composite Numbers: Have more than two factors.

The number “1” is neither prime nor composite – it has exactly one factor (“1”).

Finding the prime factorization of a number is one of the ways we can find factors. Using a factor tree is probably the most frequently known method of finding a prime factorization. In prime factorization, all of the numbers are prime and cannot be broken down anymore. As we will soon see, prime factorization can help us do many tasks in mathematics.

Example: Find the prime factorization of 12.

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  12
    /\ \
   /  \  \
 12
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Notice how it doesn’t matter which way we initially break down 12. Our final answer is the same: $2 \times 2 \times 3$ or $2^2 \times 3$. Frankly, $2 \times 2 \times 3$ is a more useful form to us than $2^2 \times 3$.

Having the prime factorization is helpful in finding all factors of a number. When finding all factors of a number, we are looking for both prime and composite factors and the number 1. Don’t forget the obvious pair of 1 and the number itself!

Example: Find all factors of 12. (Don’t forget the number “1”!)

Since 12 breaks down to $2 \times 2 \times 3$, then we know that 2 and 3 go into 12, so we have 1, 2, 3 and the numbers they pair up with, which are 12, 6, and 4.

Answer: 1, 2, 3, 4, 6, 12

Notice how 4 is $2 \times 2$ and 6 is $2 \times 3$, which are both parts of $2 \times 2 \times 3$. 
Another thing to keep in mind about prime factorization is that it helps us to rule out numbers that don’t go into a number. For example, if we break down 36 into 2 x 2 x 3 x 3, then we know that 5 doesn’t go into 36 because 5 is prime and it would have to be listed in the prime factorization if it did. The same is true for 7, 11, 13, etc.

Example: Find the prime factorization of 96 and then find all factors of 96.

96

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All factors:

Example: Find the prime factorization of 210 and then find all factors of 210.

210

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All factors:

Remember, prime factorization means to break a number down into a product of prime numbers, and finding all factors means to find every number that goes into a number.