<u>**Vectors and Statics**</u>

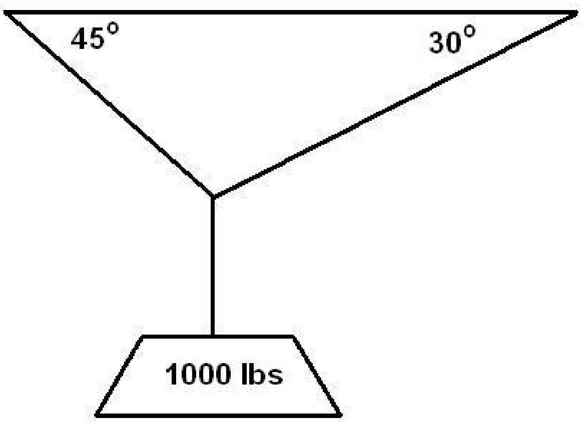
The idea of vectors can be used in an engineering concept called statics. There are two things that are true:

1. An object at rest is in what we call **static equilibrium**.
2. For an object to remain in static equilibrium, the sum of the horizontal components and the sum of the vertical components of **all** forces acting on the body must each be 0.

Examine the problem below.

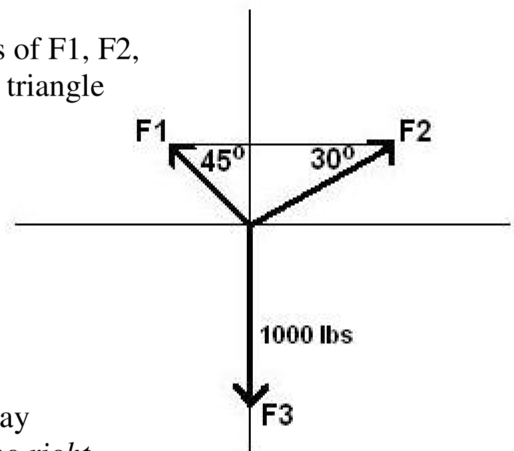
Example:

A weight of 1000 lbs is suspended from two ropes as shown. What is the tension on each rope?



The forces exerted can be drawn using vectors like so. We label Force 1 as F1 and so on.

We find the values of F1, F2, and F3 using right triangle trigonometry.



The sum of all of the horizontal components must equal 0 and the sum of all of the vertical components must also equal 0. Another way of looking at it is to say the *forces going left must equal the forces going right*, and the *forces pulling upward must equal the forces pulling downward.*

Notice that F3 pulls downward 1000 lbs, but pulls neither left nor right. For the other two angles, we have the following.

For the horizontal forces, this gives us:

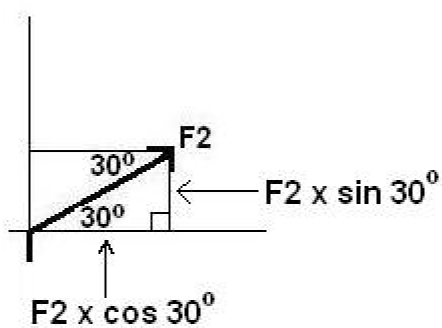
$F2 \bullet \cos 30^\circ = F1 \bullet \cos 45^\circ$

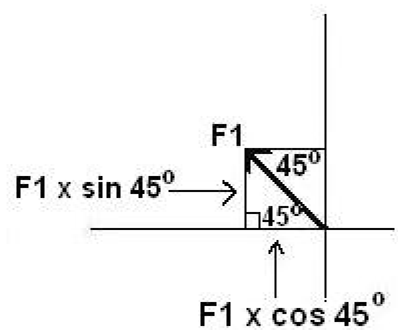
(Force left = Force right)

For the vertical forces, this gives us:

$F2 \bullet \sin 30^\circ + F1 \bullet \sin 45^\circ = 1000$

(Forces up = Force down)





If we solve for F2 in the first equation, we get $F2 = 0.8165 \bullet F1$. Then we can substitute 0.8165 • F1 in for F2 in the second equation:

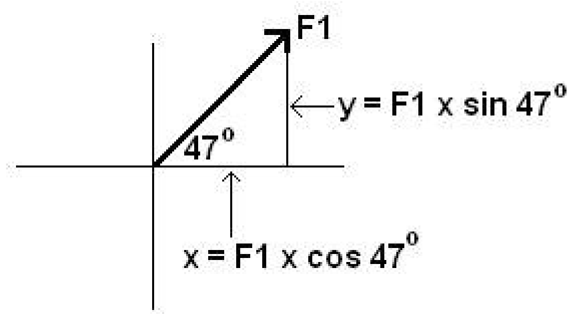
$F2 \bullet \sin 30^\circ + F1 \bullet \sin 45^\circ = 1000$

$0.8165 \bullet F1 \bullet \sin 30^\circ + F1 \bullet \sin 45^\circ = 1000$ substitute $0.8165 \bullet F1$ for F2.

$0.8165 \bullet F1 \bullet 0.5 + F1 \bullet 0.7071 = 1000$	evaluate sin 30° and sin 45°
$0.4082 \bullet F1 + F1 \bullet 0.7071 = 1000$	multiply 0.8165 by 0.5
$1.1154 \bullet F1 = 1000$	combine like terms
$\frac{1.1154 \bullet F1}{1.1154} = \frac{1000}{1.1154}$	divide both sides by 1.1154
$F1 = 896.58$ lbs	This is the magnitude of Force 1

To find F2, just substitute 896.58 in for F1 in the first equation $F2 = 0.8165 \bullet F1$.
$F2 = 0.8165 \bullet F1$

$F2 = 0.8165 \bullet 896.58 = 732.1$ lbs	This is the magnitude of Force 2.



So, why do the horizontal and vertical components of the vectors have the values they have? In other words, why is y = F1 • sin 47° and x = F1 • cos 47°?
It boils down to right triangle trigonometry.
If the hypotenuse of the right triangle has a force of F1, then for "y":

$$\sin 47^\circ = \frac{opp}{hyp} = \frac{y}{F1}.$$

$F1 \bullet \sin 47^\circ = \frac{y}{F1} \bullet F1$	Multiply both sides by F1.
$F1 \bullet \sin 47^\circ = y$	F1 cancels on the right side.

For "x":

$$\cos 47^\circ = \frac{adj}{hyp} = \frac{x}{F1}$$

$F1 \bullet \cos 47^\circ = \frac{x}{F1} \bullet F1$	Multiply both sides by F1.
$F1 \bullet \cos 47^\circ = x$	Again, F1 cancels on the right side.