Solving Quadratic Equations

There are two ways to solve a quadratic equation: the quadratic formula and by factoring. There are advantages and disadvantages to each. **In both cases, we must first get the equation in the form** \( ax^2 + bx + c = 0 \) **(get zero on one side).** Also, in both cases, these values of “x” we’re finding are the x-intercepts because we’re letting \( y = 0 \).

The **Quadratic Formula:** If \( ax^2 + bx + c = 0 \), then \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \) (where \( a \) isn’t 0).

**Advantage:** It always works.  
**Disadvantage:** Lots of places to make mistakes.

Factoring: If the quadratic can be factored, we can set \((ax + b)(cx + d) = 0\) and each piece then equals zero \( \rightarrow (ax + b) = 0 \) and \((cx + d) = 0\).

**Advantage:** Easier than the quadratic formula.  
**Disadvantage:** Doesn’t always work.

We’ve already seen where the quadratic formula can be useful, so let’s look at a situation where we can factor.

**Example 1:**

a) Solve \( x^2 – 7x = –12 \) by factoring  
b) Solve \( x^2 – 7x = –12 \) by the quadratic formula.

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**Graph of** \( y = x^2 - 7x + 12 \)

**Notice:**

1) Factoring, when possible, tends to be easier than using the quadratic formula.
2) When you use the quadratic formula, you can tell if factoring is possible because the value you get under the square root will be a nice, square number.
3) Can you see the values of \( x \) you found on the graph to the left?  
They are \((3, 0)\) and \((4, 0)\). (Where \( y = 0 \); the x-intercepts)
The disadvantage to factoring is that it doesn’t always work. What do I mean by “work”? I mean that sometimes there are x-intercepts that cannot be found by factoring because those x-intercepts are not rational numbers. But, just because factoring doesn’t work, it doesn’t mean that there are no x-intercepts. This is where the quadratic formula comes into play.

**Example 2:** Solve $x^2 + x - 3 = 0$

It’s hard to tell what the exact x-intercepts are without the quadratic formula.

How do we know for certain there are no x-intercepts for a quadratic equation? Because the quadratic formula will give us a negative underneath the square root.

**Example 3:** Solve $x^2 = -x - 1$  
(Remember, you have to get “0” on one side first!)

Notice there are no x-intercepts.