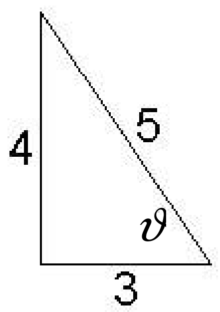
Trigonometric Ratios

We're going to examine trigonometric ratios in light of a specific right triangle and then move from there. The trig functions are sine (sin), cosine (cos), tangent (tan), cotangent (cot), cosecant (csc), and secant (sec).



Since 3-4-5 is a Pythagorean Triple, let's use that triangle. We'll define the three basic trig functions as follows:

$\sin\vartheta = \dfrac{\text{opposite}}{\text{hypotenuse}}$ $\quad\cos\vartheta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

$\tan\vartheta = \dfrac{\text{opposite}}{\text{adjacent}}$

So, for the above triangle, $\sin\vartheta = \frac{4}{5}$, $\cos\vartheta = \frac{3}{5}$, and $\tan\vartheta = \frac{4}{3}$.

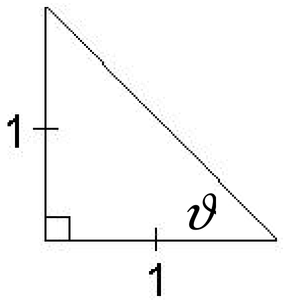
There are also the three minor trig functions.

$\cot\vartheta = \dfrac{1}{\text{tangent }\vartheta}$ $\quad\sec\vartheta = \dfrac{1}{\text{cosine }\vartheta}$ $\quad\csc\vartheta = \dfrac{1}{\text{sine }\vartheta}$

So, $\cot\vartheta = \frac{3}{4}$, $\sec\vartheta = \frac{5}{3}$, $\csc\vartheta = \frac{5}{4}$.

There are two special triangles used repeatedly in trigonometry. One is the 45-45-90 triangle and the other is the 30-60-90 triangle. Let's examine the 45-45-90 triangle first. This triangle is isosceles, where the legs are congruent.We'll let the leg lengths equal "1", find the hypotenuse, and then derive the trig ratios from there. Notice how $\vartheta = 45^0$.

1) Find the hypotenuse using the Pythagorean Theorem.



2) Now let's find the trig values for this triangle.

$\sin 45^0 = \dfrac{\text{opposite}}{\text{hypotenuse}} =$ $\cos 45^0 = \dfrac{\text{adjacent}}{\text{hypotenuse}} =$

$\tan 45^0 = \dfrac{\text{opposite}}{\text{adjacent}} =$ $\cot 45^0 = \dfrac{1}{\text{tangent } \vartheta} =$

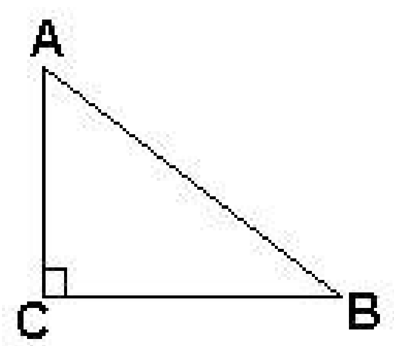
$\sec 45^0 = \dfrac{1}{\text{cosine } \vartheta} =$ $\csc 45^0 = \dfrac{1}{\text{sine } \vartheta} =$

The nifty thing about trigonometric ratios is that no matter what size the triangle, the trig values stay the same for all 45-45-90 triangles. That's because of their similarity!

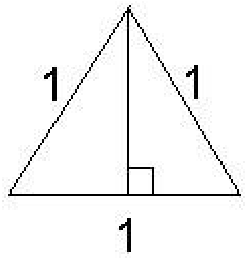
For example, let's say we have a 45-45-90 triangle with leg lengths 5.

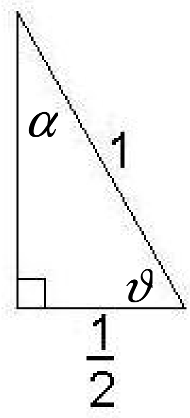
1) Find the hypotenuse.

2) Find the sine, cosine, and tangent for one of the 45^0 angles.



The other triangle that's important in trigonometry is the 30-60-90 triangle. Let's start off with an equilateral triangle, which has three 60^0 angles. If we drop down the perpendicular from the top angle, it bisects the base and also bisects the top angle. In other words, our equilateral triangle is divided into two 30-60-90 triangles with a hypotenuse of 1. The base is divided into two parts each with a length of $\frac{1}{2}$.





1) Find the length of the second leg using the Pythagorean Theorem.

2) Find the value of the six trig ratios for angle $\vartheta = 60^0$.

sin 60^0 = cos 60^0 = tan 60^0 =

csc 60^0 = sec 60^0 = cot 60^0 =

3) Find the value of the six trig ratios for angle $\alpha = 30^0$.

sin 30^0 = cos 30^0 = tan 30^0 =

csc 30^0 = sec 30^0 = cot 30^0 =

Let's add these values to our degree/radian/trig value chart.

There are other angles on our unit circle chart that yield the same triangles as our 45-45-90 triangle and our 30-60-90 triangle. We are going to examine those next.