Systems of Equations

Anytime we have two or more equations, we have what is called a system of equations. In our case, we are going to be interested in two linear equations. The goal of a system of equations is finding the place where the (x, y) value is the same for both lines, or where the lines cross. One way we can do this is by graphing.

Ex 1: \[
\begin{align*}
y &= 2x + 3 \\
y &= x + 2
\end{align*}
\]

Ex 2: \[
\begin{align*}
2x + y &= 5 \\
2x + y &= 7
\end{align*}
\]

Ex 3: \[
\begin{align*}
2x - 3y &= 8 \\
4x - 6y &= 16
\end{align*}
\]

Notice that in example 1, we get two lines that cross at a single point. In example 2, we get two lines that are parallel, and in example 3, we get a single line because the lines cross everywhere (both lines are the same). These are the three possibilities with systems of equations. Notice that in the last two examples, both lines in the system have the same slope. Graphing is a great way to get an idea of where your lines are going to cross, if at all, but sometimes it can only give us an estimate and not the exact answer. This happens when the equations cross at points that are not “nice”.

Substitution

A method called substitution can be used instead of graphing in order to get a more accurate result. This is best used when one of the coefficients of “x” or “y” in either equation is “1”. Let’s check out example 1 above using substitution.

Ex 1: \[
\begin{align*}
y &= 2x + 3 \\
y &= x + 2
\end{align*}
\]

Notice that we have solved for “y” in both equations. While doing this in both equations is not necessary, we can use it to our advantage.
2x + 3 = x + 2  Since “y” is equal to both 2x + 3 and x + 2, then they are equal to each other! Then, we can solve for “x” and plug our value back into either equation to get “y”. It doesn’t matter which, as we should get the same “y” value either way. 

\[ y = (______) + 2 = \]

So, my point, is (____, ____).

Ex 2: \[
\begin{align*}
2x + y &= 5 \\
2x + y &= 7
\end{align*}
\]

Let’s try this same idea on example 2. We can solve for “y” in either equation above, since the coefficient of “y” is “1” in both of them, so I’ll use the first: 2x + y = 5

2x + y = 5

\[ y = ________ \]

\[ \leftarrow \]

Now that we’ve found what “y” is equal to, let’s plug it in the second equation, 2x + y = 7.

\[ 2x + (______) = 7 \]

Again, distribute as necessary and solve for “x”, if possible. In this case, however, see how the “x”s cancel out? Here we ended up with a statement that is false. Therefore, there are no places where the lines cross.

Sometimes, we get a true statement instead of a false statement. In this case, the lines cross everywhere, or are the same line. This would be the case with example 3.

Using substitution on example 3 is not fun. Here is why.

\[
\begin{align*}
2x - 3y &= 8 \\
4x - 6y &= 16
\end{align*}
\]

If we solve either of the equations for either “x” or “y”, then we end up with fractions. Blar! 😞

2x - 3y = 8 is either \[ y = \frac{2}{3}x - \frac{8}{3} \] or \[ x = \frac{3}{2}y + 4 \] (and so is 4x - 6y = 16, actually).

Either way, plugging in fractions is not necessarily a good time. Luckily, there is yet another way to do these that works every time and mostly eliminates having to work with fractions.
**Elimination**

Elimination involves eliminating either the “x” or “y” variable out of both equations so the other can be solved for. Before we get into that, let’s examine a few true number sentences. We’re going to add these equations up and down to see if we get another true statement.

\[
\begin{align*}
2 + 9 &= 11 \\
3 + 5 &= 8
\end{align*}
\]

Try it with some negatives!

\[
\begin{align*}
-9 + 2 &= -7 \\
-2 - 12 &= -14
\end{align*}
\]

Adding up and down gives us another true statement! So, this is the idea we are going to use for variables.

Ex 4: \[\begin{cases} 2x + y = 5 \\ x - y = -2 \end{cases}\]

So, add these up and down. Notice how the “y” value cancels out completely. Take the value we got for “x” and now plug it into one of the original two equations. Then, solve for “y”.

\[2(\underline{5}) + y = 5\]

We can use this method for checking out example 3.

\[\begin{align*}
2x - 3y &= 8 \\
4x - 6y &= 16
\end{align*}\]

The thing here is that if we just add, then nothing will cancel out! We’ll get \(6x - 9x = 24\). So, before we go any further, we need to multiply either the top or the bottom equation by something that will make one of the variables cancel. Let’s multiply the top equation by -2. Doing so, will make the “x”s cancel: \(-2(2x - 3y = 8) \rightarrow -4x + 6y = -16\)

\[\begin{align*}
-4x + 6y &= -16 \\
4x - 6y &= 16
\end{align*}\]

\[\text{So, here is what we now have. We can add these now! See how everything cancels out? That means these are the same line.}\]