

Lines and their Slopes

The slope of a line is a ratio that measures the incline of the line. There are several things to notice about slopes:

- 1) The smaller the incline, the closer the slope is to zero and the steeper the incline, the farther the slope value is away from zero.
- 2) The slope is the same value from point to point (or between any two points) on the line.
- 3) A positive slope means that the line is increasing, or going uphill, from left to right. In other words, as “x” gets bigger, “y” also gets bigger.
- 4) A negative slope means that the line is decreasing, or going downhill, from left to right. In other words, as “x” gets bigger, “y” gets smaller.

The slope ratio is calculated one of three different ways.

- 1) Using the graph: $m = \frac{\text{rise}}{\text{run}}$, where the “rise” is how much y changes from one point to another and the “run” is how much x changes from one point to another.
- 2) The slope formula, which is usually stated like so: Given two points (x_1, y_1) and (x_2, y_2) , $m = \frac{y_2 - y_1}{x_2 - x_1}$.
- 3) Calculate how much y changes from point to point and divide that by how much x changes from point to point. This method requires a little more explanation.

Let’s say we’re given the points $(3, 6)$ and $(5, 9)$ and we want to find what the slope of the line is between them. I’m going to make a “t” chart. In the chart below, “ Δx ” means “change in x” and “ Δy ” means “change in y” (how much “x” and “y” change from point to point).

Δx	x	y	Δy
+2	3	6	+3
	5	9	

x increases by 2 \longrightarrow \longleftarrow y increases by 3

So, my slope is $\frac{\Delta y}{\Delta x} = \frac{+3}{+2} = \frac{3}{2}$.

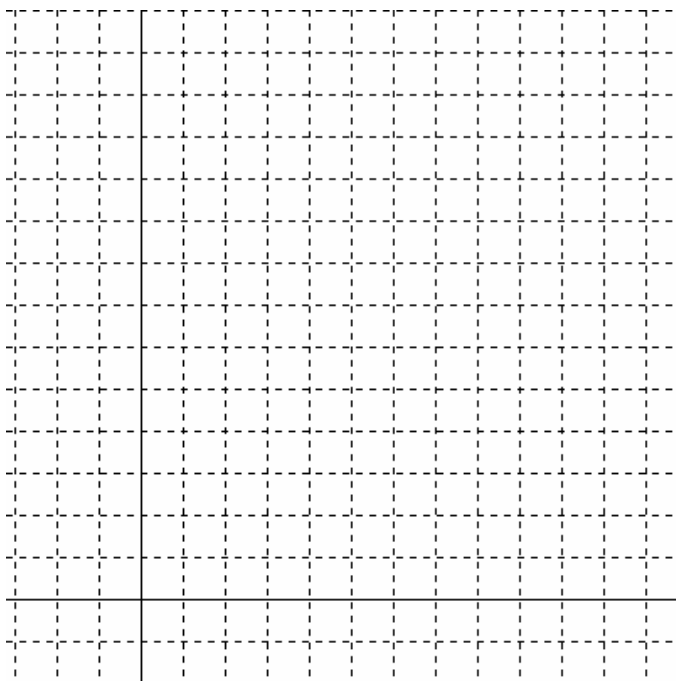
Let's use the slope formula on these same two points:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(\quad) - (\quad)}{(\quad) - (\quad)} = \underline{\hspace{2cm}}. \quad \text{See how we get the same thing?}$$

Does it matter which point we put first? Let's try it and see!

$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{(\quad) - (\quad)}{(\quad) - (\quad)} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}.$$

Let's plot these two points and see if we can see the slope as $\frac{\text{rise}}{\text{run}}$



x	y
3	6
5	9

We can also use our Δx and Δy to find another point on the line. We just use the $\Delta x = +2$ and the $\Delta y = +3$ to get that point.

Let's find the slopes of the lines between a few pairs of points.

- 1) (-2, -7) and (1, -1) 2) (2, -3) and (-1, 4) 3) (0, 3) and (-5, 4)

x	y
-2	-7
1	-1

x	y
2	-3
-1	4

x	y
0	3
-5	4

We can also examine the slope of a line based on the given equation of the line. For example, let $y = \frac{3}{4}x + 2$. Let's plug in some values for "x" and see what we get for "y".

Δx	x	y	Δy
	0		
	4		
	8		

Let's check the Δx and Δy .

$$\frac{\Delta y}{\Delta x} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$$

Do you see this number represented in the equation of the line?

Another thing to notice is that when $x = 0$, the y-value is 2. Do you also see the number "2" represented in the equation $y = \frac{3}{4}x + 2$? This is because when we plug in "0" for x, the "mx" portion of the equation disappears and we're left with $y = b$.

This slope and the point (0, 2) demonstrate why $y = mx + b$ is called the "slope-intercept form". The coefficient of "x" is the slope and the "b" is the y-intercept (the y-value when $x = 0$ – this is where the line crosses the y-axis).

Let's examine a few lines and see if we can find the slope and a point on the line.

1) $y = \frac{2}{5}x - 6$ 2) $y = 4x + 3$ 3) $y = 10 - \frac{1}{8}x$ 4) $y = x - 7$ 5) $y = 15 + 3x$

m = _____ m = _____ m = _____ m = _____ m = _____
 point _____ point _____ point _____ point _____ point _____

If we are given a line in standard form ($Ax + By = C$), one way we can find the slope is simply by solving for y and identifying the coefficient of x.

Solve the following lines for y and identify the slope and y-intercept coordinates.

1) $6x + y = 18$ 2) $2x + 5y = 10$ 3) $3x - 2y = 12$

m = _____ m = _____ m = _____
 point = _____ point = _____ point = _____