Properties of Numbers

In mathematics, there are several properties that we commonly take for granted, but that are important in the workings of our number system. The three we use most frequently are the associative property, the commutative property, and the distributive property.

The Associative Property – The Associative Property allows us to rearrange the order of working certain numbers so that problems are easier to answer. It says:

$$(a + b) + c = a + (b + c) \quad \text{and}$$
$$(a \cdot b) \cdot c = a \cdot (b \cdot c)$$

This means that if we have (19 + 3) + 7, we can write it as 19 + (3 + 7) to get "10" in the parentheses, which makes it easier to add. (Sometimes we call this "making tens".)

It also means if we have (14 • 2) • 5, we can write it as 14 • (2 • 5) to get 14 • 10, or 140, which is easier to do in our heads than 28 • 5.

Notice that the associative property is only defined for adding and multiplying, not subtraction [$5 - (7 - 10) \neq (5 - 7) - 10$] or division [$12 \div (6 \div 2) \neq (12 \div 6) \div 2$].

The Commutative Property – The Commutative Property allows us to actually reverse numbers if we need to. It says:

$$a + b = b + a \quad \text{and}$$
$$a \cdot b = b \cdot a$$

As an example of the usefulness of this property, think about 3 • 4 and 4 • 3.

3 • 4 actually means "3 groups of 4" or "4 + 4 + 4". (3 kids get 4 cookies each)

4 • 3 actually means "4 groups of 3" or 3 + 3 + 3 + 3. (4 kids get 3 cookies each)

The Commutative Property tells us that 3 • 4 = 4 • 3, even though the two actually represent two different sets, and we know that both equal 12.

It also means that if we have 3 + 32 + 7, we can rewrite this 3 + 7 + 32 (the 32 and 7 switch places) and then get our 10 to add to 32.

Again, notice that this isn't necessarily true for subtraction ($3 - 5 \neq 5 - 3$) or division ($6 \div 3 \neq 3 \div 6$).

The Distributive Property – The Distributive Property is especially useful in algebra in simplifying expressions. It says:

$$a \cdot (b + c) = a \cdot b + a \cdot c$$

Look at 2 • (3 + 4). We would work this by doing what is inside parentheses first and then multiplying: 2 • (3 + 4) = 2 • (7) = 14.

The Distributive Property says we can do this instead: 2 • 3 + 2 • 4 = 6 + 8 = 14.

It doesn't seem to make math easier, but in a problem with a variable, there's really no other way to work it.

Example: $3(x + 2) = (x + 2) + (x + 2) + (x + 2) = 3x + 6$ can also be written like so:

$$3(x + 2) = 3x + 6$$

The Distributive Property works regardless of the number outside of the parentheses. It also works if the numbers inside of the parentheses are negative. In other words, $2(x – 4) = (x – 4) + (x – 4) = 2x – 8$ or

$$2(x – 4) = 2x – 8$$

Let's try some problems! Multiply the following:

1) $5(x + 6)$ 2) $-6(x + 3y – 4)$ 3) $-\frac{1}{2}(4x + 14)$

Factoring – Factoring is just using the Distributive Property in the opposite way of multiplying. In factoring, we are looking to see what number is in common to all terms in the expression so that we can "factor it out".

Example: Factor $4x – 16$. What is the largest number that will go into both 4 and 16? ______ Pull it outside, and then see what is left on the inside of the parentheses. Remember, when we multiply back through, we need to make sure we get what we started with: ____(   )

Practice! Factor the following!

1) $6x + 9$ 2) $12a – 18b + 48c$ 3) $24 – 6m$