Polynomial Roots

Remember back when we were talking about using either factoring or the quadratic formula to find the x-intercepts, or roots, of a quadratic equation. Other polynomial equations also have roots. There are various ways to find these roots. Most of these, such as the “cubic formula” (like the quadratic formula), we will not concern ourselves with because it is long and messy. For our purposes, we are going to concern ourselves with roots that we can find by factoring or using the quadratic formula. We are also going to take roots that are given to us and find the equation of the polynomial based on that information.

The first thing we’re going to do is find polynomial roots. Like with quadratics, we must get “0” on one side by itself first.

Example 1  Solve for x:  \[ x^3 = x \]  \[ \Rightarrow \text{Get “0” on one side!} \]

\[ \frac{-x}{-x} \]

\[ x^3 - x = 0 \]  \[ \Rightarrow \text{Now, factor!} \]

\[ \text{____(_____)(_____)} = 0 \]

\[ \text{____} = 0 \]

\[ \text{____} = 0 \]

\[ \text{____} = 0 \]

\[ \Rightarrow \text{Set each part equal to “0”}. \]

What we end up with is three values of “\(x\)”, all of which will give us \(y = 0\) if we plug them in.

In other words, \((____, 0)\), \((____, 0)\), \((____, 0)\) are all on the graph of

\[ f(x) = x^3 - x \].

Let’s graph the function. Remember, the coefficient “\(a\)” is positive and the equation is cubic.

Example 2  Solve for x:  \[ x^3 - x^2 - 6x = 0 \]  \[ \Rightarrow \text{“0” is already on one side.} \]

\[ \text{____(_____)} = 0 \]

\[ \text{____(_____)(_____)} = 0 \]

\[ \text{____} = 0 \]

\[ \text{____} = 0 \]

\[ \text{____} = 0 \]

\[ \Rightarrow \text{Set each part equal to “0”}. \]
So, the x-values we get for \( y = x^3 - x^2 - 6x \) when \( y = 0 \) are ___, ___, and ___.
The points on the graph are (____, 0), (____, 0), (____, 0).

“a” is ___________
the equation is _______.

There are also quadratic-type equations that are similar to quadratics in form, but not exactly quadratic. We’re familiar with \( x^2 + 3x + 2 \), but a quadratic-type would be \( x^4 + 3x^2 + 2 \) or \( x^6 + 3x^3 + 2 \).
In all of the cases of quadratic-type equations, the middle term has a variable with exponent \textit{half} of the degree of the equation.

\textbf{Example 3} Solve for \( x \): \( x^4 - 5x^2 + 4 = 0 \)

Here we get four points: (____, 0), (____, 0), (____, 0), (____, 0). So, how is the graph going to look?

“a” is ___________
the equation is __________.