Percentages and Proportions

One of the methods we discussed for solving percentage problems is by using a proportion in the following format: \( \frac{\%}{100} = \frac{\text{part}}{\text{whole}} \) The left side is the definition of percent as “per hundred” and the right side is the definition of a fraction. We can be a little more specific with this format when it comes to solving percentage word problems.

The idea here is that, like with a proportion, we have to keep alike terms across from alike terms both on the top or the bottom. Here’s the specific set-up:

\[
\frac{\% \text{ we're interested in}}{\% \text{ of the whole (100)}} = \frac{\text{amount we are interested in}}{\text{original amount that makes a whole}}
\]

Do you see how the tops are both parts we’re interested in and how the bottoms are both wholes? Do you also notice that the left fraction deals exclusively with the percents and the right side deals with the numeric quantities? This helps on problems where certain values are not immediately clear.

**Example1:** A $50 pair of jeans is on sale for $35. What percent is the discount on the jeans? We’re looking for the percent of the discount, so here’s the set-up:

\[
\frac{\% \text{ of the discount}}{100} = \frac{\text{amount of the discount}}{\text{original amount}}
\]

The percentage is “x”, since that’s what we’re trying to find.

\[
\frac{x}{100} = \frac{\text{amount of the discount}}{\text{original amount}}
\]

Now we need the amount of the original and the amount of the discount.

Amount of discount = __________________ Amount of original = __________________

*Notice that the amount of the discount is not the $35. We have to find the discount amount!*
Example 2: The number of students that attended a school in Fall 2001 was 4,527. The number of students that attended the same school in Fall 2002 was 5,071. What was the percent of increase in the school population from Fall 2001 to Fall 2002?

\[
\frac{\text{amount of increase}}{\text{original amount}} = \frac{\text{of the increase}}{100} = \frac{\text{of the increase}}{\text{original amount}}
\]

Again, we’re trying to find the percent of increase, so that’s “\(x\)”. For the rest:

\[
\text{Amount of increase} = \underline{\hspace{2cm}} \quad \text{Amount of original} = \underline{\hspace{2cm}}
\]

\[
\frac{x}{100} = \underline{\hspace{2cm}}
\]

Example 3: Over the next five years, tuition is expected to increase 5%, or $100 a semester, over what it is now. What is the tuition now?

\[
\frac{\text{amount of increase}}{\text{original amount}} = \frac{\text{of the increase}}{100} = \frac{\text{of the increase}}{\text{original amount}}
\]

This time, we know the percent of the increase (5%) and the amount of the increase ($100) and we want to know the original tuition amount (\(x\)).

\[
\underline{\hspace{1cm}} = \underline{\hspace{1cm}}
\]

100 = \(x\)

Example 4: A bottle of stain remover claims it contains 25% more product free over its original packaging. The new packaging contains 10 fl. oz. of stain remover. What was the original amount it held?

This one’s a little different because we can’t tell what the amount of increase is without knowing the original. But, here’s what we do know: the 10 fl. oz. is \(\underline{\hspace{1cm}}\)% of the original amount. So, we can do this, where the original amount is what we’re looking for:

\[
\frac{\text{of new amount}}{100} = \frac{\text{new amount}}{\text{original amount}}
\]

\[
\underline{\hspace{1cm}} = \underline{\hspace{1cm}}
\]

100 = \(x\)