Exponents

Dealing with positive and negative exponents and simplifying expressions dealing with them is simply a matter of remembering what the definition of an exponent is.

- A positive exponent means repeated multiplication.
- A negative exponent means the *opposite* of repeated multiplication, which is repeated division.

Let’s evaluate the following:

\[ 2^3 = \]
\[ 2^2 = \]
\[ 2^1 = \]
\[ 2^0 = \]
\[ 2^{-1} = \]
\[ 2^{-2} = \]
\[ 2^{-3} = \]

Notice that a negative exponent makes the “2” flip from $\frac{2}{1}$ raised to some power over to $\frac{1}{2}$ raised to some power. In other words, instead of multiplying by 2 repeatedly, we’re dividing by two repeatedly. In effect, a negative exponent makes us do the opposite of what it initially looks like. If a number with a negative exponent is in the bottom (division), it means to move it to the top (multiply); If a number with a negative exponent is in the top (multiplication), it means to move it to the bottom (divide).

Simplify:

1) $x^{-5}$
2) $\frac{1}{x^{-6}}$
3) $5x^{-4}$
4) $\frac{5}{6x^{-2}}$
5) $(3x)^{-2}$
Notice that only whatever is directly next to the exponent is affected by the exponent. In #3 and #4, the numbers 5 and 6 are not moved or otherwise changed. In #5, the entire parenthesis is affected by the exponent.

**Laws of Exponents**

At this point, the laws of exponents should be fairly well known to us. We’re going to examine where these “laws” came from.

Example 1: Simplify: $x^2 \cdot x^4$  

Example 2: Simplify $\frac{x^2}{x^6}$

Example 3: Simplify $(x^3)^4$  

Example 4: Simplify $\left(\frac{x^4}{x^3}\right)^2$

When simplifying an expression involving positive and negative exponents, it’s sometimes easier to move the base with the negative exponents first. That way, when we use the “laws of exponents” that we all know, we don’t have to worry about double negatives and the like.

Example 5: Simplify $\frac{x^{-4}}{x^5}$  

Example 6: Simplify $(x^{-3}y^2)^4$
A rational (or fractional) exponent is the combination between a power and a root. Normally, $x^3$ is the same as $x^{\frac{3}{1}}$. Notice that the denominator is “1”, so there’s no root. In something like $x^{\frac{3}{2}}$, the “3” still means the power that “x” is being raised to, but the “2” represents that we also have to take the square root.

$$x^{\frac{3}{2}} = \sqrt{x^3}$$

Do you think it matters whether we take the root of $x$ first or cube the $x$ first?

Try this by taking the root first and then squaring: $8^{\frac{2}{3}} \rightarrow (\sqrt[3]{8})^2 = (\text{____})^2 = \text{____}$

Now try the same problem by squaring first and then taking the cube root:

$8^{\frac{2}{3}} \rightarrow \sqrt[3]{8^2} = \sqrt[3]{\text{____}} = \text{____}$

So, does it matter which we do first? ______

Let’s practice a few of these. Remember what the negative exponent means!

1) $16^{\frac{3}{2}}$  
2) $25^{\frac{1}{2}}$  
3) $64^{\frac{-2}{3}}$  
4) $-8^{\frac{5}{3}}$