

Multiplying Binomials and Factoring Quadratics

Multiply the following:

$$1. (x + 3)(x + 5)$$

$$\begin{array}{cccc} _ & + & _ & + & _ & + & _ \\ _ & + & _ & + & _ & & \end{array}$$

$$2. (x + 2)(x - 6)$$

$$\begin{array}{cccc} _ & + & _ & + & _ & + & _ \\ _ & + & _ & + & _ & & \end{array}$$

$$3. (x - 4)(x - 7)$$

$$\begin{array}{cccc} _ & + & _ & + & _ & + & _ \\ _ & + & _ & + & _ & & \end{array}$$

In general, let's say you're given $(x + a)(x + b)$.

$$\text{Multiply: } (x + a)(x + b) = _ + _ + _ + _ = _ + (_)x + _$$

- The first term is always _____.
- The coefficient of the first term is _____.
- The coefficient of the middle term is _____.
- The last term (or constant) is _____.

The purpose here is for you to notice that the coefficient of "x" (the middle term) is the sum of "a" and "b" and the constant (the last term) is the product of "a" and "b". *This is only true if the coefficient of the first term is 1.* This helps when we're trying to factor.

For example, let's factor $x^2 + 5x + 6$. We want it to be in the form of $(x + a)(x + b)$.

Remember:

- The product of "a" and "b" is 6.
- The sum of "a" and "b" is 5.

Factors of 6	
1	6
-1	-6
2	3
-2	-3

So, we're looking for factors of 6 that add to be 5. →

Note that since $2 + 3 = 5$, we can "break up" the middle term of "5x" like so:

$$\begin{array}{c} x^2 + 5x + 6 \\ x^2 + 3x + 2x + 6 \end{array}$$

And group together the first two terms and the last two terms:

$$\underline{x^2 + 3x} + \underline{2x + 6}$$

Factor out what you can in each grouping:

$$x(x + 3) + 2(x + 3)$$

Factor $(x + 3)$ out of each grouping:

$$(x + 3)(x + 2)$$

This may not seem easier than any way you learned how to factor previously, but this method works easier than “trial and error” if your coefficient of “ x^2 ” isn’t 1. In other words, this works for factoring things like $2x^2 + 5x + 3$.

The only difference is because the coefficient of “ x^2 ” isn’t 1, but instead is 2, you’ll be looking for factors of the product of 2 and 3 whose sum is 5.

Factors of $2 \times 3 = 6$	
1	6
-1	-6
2	3
-2	-3

Since $2 + 3 = 5$, then these are the factors that need to be used. →

So:

$$2x^2 + 5x + 3$$

$$2x^2 + 2x + 3x + 3$$

Group together like previously:

$$\underline{2x^2 + 2x} + \underline{3x + 3}$$

Factor:

$$2x(x + 1) + 3(x + 1)$$

$$(x + 1)(2x + 3)$$

See? Much easier than the “trial and error” method! Try it for these:

1. $3x^2 + 17x - 6$

$$3x^2 + \underline{\quad} + \underline{\quad} - 6$$

$$\underline{\quad} + \underline{\quad}$$

$$\underline{\quad}(\underline{\quad}) + \underline{\quad}(\underline{\quad})$$

$$(\underline{\quad})(\underline{\quad})$$

$$\underline{3x - 6}$$

2. $12x^2 - 7x - 12$

$$12x^2 + \underline{\quad} + \underline{\quad} - 12$$

$$\underline{\quad} + \underline{\quad}$$

$$\underline{\quad}(\underline{\quad}) + \underline{\quad}(\underline{\quad})$$

$$(\underline{\quad})(\underline{\quad})$$

$$\underline{12x - 12}$$