

Factoring

Factoring is just another way of looking at the distributive property. In multiplication, we can “distribute through” a number or variable to all of the inside numbers/variables like so: $3x(2x^2 + 4x + 5) = 6x^3 + 12x^2 + 15x$. But, one thing to keep in mind is that if we can forward, then we can also go backwards! In other words, we can go from $6x^3 + 12x^2 + 15x$ to $3x(2x^2 + 4x + 5)$. That’s called “factoring”.

In factoring out a monomial (or greatest common factor), I usually will break my problem down into parts. Let’s check out this problem.

$$\text{Factor } 4x^3 + 12x^2 - 8x^5.$$

The first part we look at will be the coefficient. What is the largest number that will go into 4, 12, and 8? _____

The second thing we look at is the variable. If there is more than one variable, we look at them individually. What is the most number of x’s that we can take out from all three parts of $4x^3 + 12x^2 - 8x^5$?

Keep in mind that while the first monomial $4x^3$ has three x’s, the $12x^2$ only has two x’s, so we can’t take out three. The $12x^2$ just doesn’t have three available to give up. In fact and in general, we can only take out what the least number of x’s are in the monomials present. In this case, how many x’s can be taken out? _____

After we figure out what’s going to be taken out, we have to see what’s going to be left inside the parentheses. Actually, we’re dividing when we factor. Remember, when we multiply back through, we’d better get what we started with! In this case, we started out with three monomials, so we should have three monomials left inside the parentheses.

Okay, let’s factor: $4x^3 + 12x^2 - 8x^5 = \underline{\hspace{1cm}}(\underline{\hspace{1cm}} \underline{\hspace{1cm}} \underline{\hspace{1cm}})$

Let’s try several more on the next page.

1) $12x^4 - 18x^8 + 36x^6$

2) $6x^2y^5 - 9x^4y^3 - 21x^6y^2$

3) $2xy - 6x^2y^3 + 10x^3y$

4) $4ab - 8a^3b^6$

*Note: when you factor out an entire monomial, we must leave a placeholder because we are actually dividing. That placeholder is "1".

Also, it turns out that it doesn't matter what the common factor is in all parts. As long as they are identical and present in each monomial, then they can be factored out! It could even be a star or a heart or anything else!

5) $x♥ + 7♥$

6) $x★ + 7★$

7) $x☀ + 7☀$

Notice the similarities in the answers to each of these factored problems. Now, expand this idea to parentheses and groups of numbers inside of parentheses.

8) $x(\square) + 7(\square)$

9) $x(a + b) + 7(a + b)$

10) $x(x - 2) + 7(x - 2)$

See how the parentheses, once they are factored out, only show up once in our factored answer? Again, notice that the values inside the parentheses are equal. Let's try more!

11) $x(x - 5) + 9(x - 5)$

12) $x(x + 11) + 15(x + 11)$

13) $3x(x^2 + 4) - 4(x^2 + 4)$

14) $x^2(x^3 + 3) - 5(x^3 + 3)$

Factoring by Grouping

Factoring by grouping gets its name because we're going to group terms together to factor them. An example of a problem where we use factoring by grouping is this:

$$2x^3 - 4x^2 + 3x - 6.$$

Notice that it has four terms. We're going to group the first two terms and group the last two terms together. It's not recommended that we put parentheses around our groupings because that can potentially mess up our equation if there's a negative involved, so I'm just going to underline them:

$$\underline{2x^3 - 4x^2} + \underline{3x - 6}$$

The idea is to factor out everything we can in the first grouping, which is _____. Then, factor out everything we can in second grouping, which is _____.

*Note – in the second grouping, take the sign of the third term in the factoring. In this case, we'd factor out a positive because of the +3x.

$$\underline{2x^3 - 4x^2} + \underline{3x - 6} = \text{____}(\text{_____}) + \text{____}(\text{_____}).$$

The factoring by grouping problem is now in that format that we examined earlier. We have $2x^2$ (something) + 3 (that same something). No, matter what that "something" is, we can factor it out, as long as it's identical. So, we're factoring out the $(x - 2)$. Then we have to see what's left over after we factor that $(x - 2)$ out.

$$2x^2(x - 2) + 3(x - 2) = (x - 2)(\text{_____})$$

One thing that's important to note is the fact that when we have an expression where the **product of the first and last terms = the product of the middle two terms, then we can use factoring by grouping.

Keep in mind that just because sometimes the product of the first and last don't equal the product of the middle two terms of some expressions doesn't mean that they can't be factored by grouping, just that if the products do equal, then it can be factored that way. This is because sometimes we can rearrange terms to make it work out. **

This is actually going to be important later on when we start looking at factoring trinomials by something called the A-C Method.

Let's look at our example problem: $2x^3 - 4x^2 + 3x - 6$

First term x Last term = _____ 2nd term x 3rd term = _____

See how they are equal? That means that factoring by grouping will work out, which we already know.

Let's check out what happens sometimes when the product of the first and last terms is not equal to the product of the two middle terms. Take our original problem and make one little adjustment: change one of the signs.

Factor $2x^3 - 4x^2 + 3x + 6$

Let's try to factor it anyways, just to see what happens:

$$\underline{2x^3 - 4x^2} + \underline{3x + 6}$$

$$2x^2(x - 2) + 3(x + 2)$$

↑ ↑

The numbers in parentheses aren't the same, so we can't factor anything out!

Some more examples:

1) $4x^3 - 7x^2 + 8x - 14$

1st term x last term = _____

2nd term x 3rd term = _____

2) $8x^3 - 3x^2 - 16x + 6$

1st term x last term = _____

2nd term x 3rd term = _____

3) $6x^3 + 3x^2 - 4x - 2$

1st term x last term = _____

2nd term x 3rd term = _____

4) $3a - 6b + 7a^2 - 14ab$

1st term x last term = _____

2nd term x 3rd term = _____