

Dividing Decimals

The rule for dividing decimals is commonly known. We know that we must have a whole number on the outside of our division symbol (in the denominator, if we write it as a fraction) and that we end up moving our decimal point in order to get that whole number. But, why does it work the way it does?

If we go back and look at our division problem in terms of fractions, we can see what's happening. (Notice that the denominator goes outside the division symbol!)

$$0.6 \overline{)1.2} = \frac{1.2}{0.6} \quad \leftarrow \text{The problem with this is that 0.6 isn't a whole number.}$$

The way we can make this a whole number is by multiplying by 10. Why 10?

Because 0.6 is **tenths**, so we multiply by **ten**. But, remember, what we do to the bottom of a fraction, we also have to do to the top of that fraction:

$$\frac{1.2 \times 10}{0.6 \times 10} = \frac{12}{6} \quad \text{Now we can divide: } 6 \overline{)12}$$

Be careful here because sometimes folks have a tendency to move the decimal points a different number of places in the top than they do in the bottom. It **must be** the same!

$$\text{Ex: } 0.3 \overline{)2.643} = \frac{2.643 \times 10}{0.3 \times 10} = \frac{26.43}{3} = 3 \overline{)26.43}$$

See how the decimal point moved one place to the right for **both** numbers?

Now, what happens if we run out of numbers under the division symbol before we're finished dividing? We're going to have to put in 0s! Sometimes, we just don't know how many 0s to put in until we get there, so we can put them in one at a time, if we want to.

$$0.4 \overline{)1.4} = \frac{1.4 \times 10}{0.4 \times 10} = \frac{14}{4} = 4 \overline{)14.0}$$

A rational number will divide out one of two ways: as a terminating decimal (one that ends) or as a repeating decimal (one that keeps going infinitely).

Ex. of terminating decimal: $\frac{1}{2} = 0.5$ **Ex. of repeating decimal:** $\frac{1}{3} = 0.333\dots$

Repeating decimals can be written two different ways. We can put three dots to signify the decimal continues in that same form (as above), or we can put a bar over the top of the part that repeats: $\frac{1}{3} = 0.\overline{3}$

Be careful with the bar over the top. It goes **only** over the part that repeats!

What do the following mean? $0.4\overline{5}$ $0.\overline{45}$

Sometimes, we really have to go out pretty far to find where a number repeats, like in $\frac{1}{7}$. Divide this out and see what you get.

A few other nifty things about rational numbers:

- If the denominator (number outside the division symbol) is made up of 2s and/or 5s only, then it **will** terminate at some point.

Ex: $\frac{17}{40} = 40 \overline{)17.00}$ ← 40 is $2 \times 2 \times 2 \times 5$ (made up of 2s and 5s only)

- If the denominator is made up of numbers other than 2s and/or 5s and is in simplest form (completely reduced), then the only way it will terminate is if it does so before we have to start annexing 0s on under the division symbol.

Ex: $\frac{1}{6} = 6 \overline{)1.000}$ ← Repeats **Ex:** $\frac{2.37}{3} = 3 \overline{)2.37}$ ← Terminates

Some problems ask for us to round answers to the nearest hundredth or tenth. In order to do this, we have to remember to divide out to one extra place so that we know how to round.

Divide and round to the nearest hundredth: $1.3 \overline{)5.23}$

(We'll need to go out to thousandths.)